MATH 1B FINAL REVIEW

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CHAPTER 7

Some Problems:

Evaluate the following integrals, or prove they are divergent:

1. $\int_1^\infty \frac{dx}{\sqrt{x^4+\sin^2(x)}}$
2. $\int \frac{dx}{e^{2x}+4e^x+8}$
3. $\int \cot(x)\ln(\sin(x))dx$
4. $\int (2x^2+1)e^{x^2}dx$
5. $\int [1+\cos(\theta)]^2 d\theta$
6. $\int_{\pi/4}^{3\pi/4} \tan(x)dx$
7. $\int_1^\infty \frac{8}{x(x+1)^2}dx$
8. $\int_0^3 \frac{dx}{\sqrt{x}+x\sqrt{x}}$
9. $\int \frac{e^{1/x}}{x^3}$
10. $\int \frac{e^{\tan^{-1}(y)}}{1+y^2}dy$
11. $\int \frac{\sin^3 x}{\cos x}$
12. How large do we need to choose n in order to get a Simpsons Rule approximation to $\int_0^1 e^{x^2}dx$ accurate to within 0.001?

Conceptual Tasks:

(a) Come up with flowchart or checklist for dealing with generic heinous integrals (Reading section 7.5 may help!)

(b) Come up with examples of improper integrals of types 1 and 2 which converge. Come up with examples of improper integrals of types 1 and 2 which diverge.

Topics and sample book problems:

1. Integration by parts and u-substitutions (you theoretically mastered this in your previous calculus course. If not, never a better time to do so!) 7.5# 69, 7.1 #33, 7.1# 30, 7.1#65
2. Trig Integrals 7.2 # 49, 7.2# 43

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3. Trig Substitutions 7.3 #15, 7.3# 25
4. Partial Fractions
   (a) Polynomial long division 7.4 # 34, 7.4 # 21,
   (b) How do you choose your partial fractions expansion? linear factors, irreducible quadratic factors; repeated and non-repeated factors. 7.4 #43, 7.4# 47, 7.4 # 51
5. Approximate Integration
   (a) Midpoint, Trapezoid, Simpsons Rule 7.7 #6
   (b) Error formulas 7.7 # 46
6. Improper Integrals 7.8 #26, 7.8#3, 7.8 #59, 7.8 #51
   (a) Types of improper integrals
   (b) tests for convergence: p-test, comparison test

More Problems: Go to the Chapter 7 Review Section (518-519)or to Section 7.5 (488-99). First, *classify all problems by what strategy you would use to evaluate them.* Then go back and actually evaluate as many odd numbered problems as you have time for.

CHAPTER 8

Some Problems:

1. Explain how to set up a calculation to find the hydrostatic force on a submerged object.
2. Explain what a moment is for a two-dimensional system of masses, and use the x- and $y-$ moments find a formula for the center of mass of the system.
3. Consider the function $y = x^2$. Draw picture of and write the formula for the surface of revolution generated by revolving it about
   (a) the $x-$ axis between $x = 0$ and $x = 2$.
   (b) the $y-$ axis between $y = 0$ and $y = 2$.

Conceptual Tasks:

1. Derive the arc length formula.
2. How is it different to set up a problem for a surface of revolution of a curve about the $x-$ versus the $y-$axis? How about if you are asked to find the arc length of this curve?
3. What is a centroid?

Topics and sample book problems

1. Arc Length 8.1 #13, 8.1 # 16
2. Surfaces of revolution 8.2 # 11, 8.2 # 25,
3. Applications
   (a) Hydrostatic Forces 8.3# 9
   (b) Centroids/Moments 8.3 # 41, 8.3 #31

CHAPTER 11

Some Problems:
Determine if the following converge or diverge:

1. $\sum_{n=1}^{\infty}(-1)^n \tan^{-1}\left(\frac{1}{n}\right)$
2. $^* \sum_{n=0}^{\infty}\left(\frac{1}{1+3\cdot(-1)^n}\right)^n$
3. $\sum_{n=1}^{\infty}\frac{\sin(1/n)}{n^{0.3}}$
4. $\sum_{n=1}^{\infty} ne^{-n^2}$
5. $\sum_{n=2}^{\infty}\frac{n}{(\ln(n))^{\ln(n)}}$
6. $\sum_{n=1}^{\infty}\left(\frac{n}{n+1}\right)^{n^3}$
7. $\sum_{n=1}^{\infty}\frac{1}{n^{1+1/n}}$
8. $\sum_{n=1}^{\infty}(-1)^n\left(\sin(1/n^2)\right)^{1/3}$
9. $\sum_{n=1}^{\infty}\frac{\sqrt{n+1}-\sqrt{n-1}}{n}$
10. $\sum_{n=1}^{\infty}\frac{(-1)^n\sqrt{n}}{n+5}$
11. $\sum_{n=1}^{\infty}(-1)^n\sqrt{1-cos(\frac{1}{n})}$
12. $\sum_{n=1}^{\infty}\frac{(-1)^n n}{(n^3+3n)^{1/3}}$
13. $\sum_{n=1}^{\infty} n\sin(n^{-5/2}$

15. Find the interval of convergence, (think about endpoints!) of the series

$$\sum_{n=2}^{\infty}\frac{1}{\ln n}\left(\frac{x-3}{2}\right)^{2n-1}$$

16. Find the sum of $\sum_{n=1}^{\infty}\frac{(-1)^n\pi^{2n}}{n!}$
17. Find the sum of the series $\sum_{n=0}^{\infty}\frac{(-1)^{n+1}(n+1)}{4^n}$

Some Taylor Series problems:

1. Find a the first 3 terms of a Taylor expansion about $a = 2$ for $f(x) = x^2 + 2x + 7$

2. Determine the radius of convergence for series $f(x) = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{1\cdot 3\cdot 5\cdots(2n+1)}$ and show $y = f(x)$ is a solution of the differential equation $y' = 1 + xy$
3. $a = 2$ for $f(x) = x^2 + 2x + 7$ State the interval of convergence and estimate the error using Taylor's Theorem.
4. Use Taylor series to evaluate the limit $\lim_{x\to 0}\left(\frac{1-x^4-\cos(\sqrt{2}x^2)}{2x^8}\right)$

Conceptual Tasks:

1. What is the difference between a series and a sequence?
2. What does it mean for an infinite sequence or series to converge?
3. Write down a math a mathematical statement to explain the idea that "you can think of an infinite series as a sequence by considering the sequence of partial sums."
4. Come up with a flowchart or checklist for how to test for divergence or convergence of a sequence or series.
5. Name a test that only tells you about divergence (not convergence) and an example of a series it fails to give you any information about. Name a test that only tells you about convergence (not divergence) and an example of a series it fails to give you any information about.
6. What is a power series? Is $x^{-2} + 2x^{-1} + x^5 + x^{10}$ a power series?
7. What are the possibilities for the regions where a power series can converge?

Topics and sample book problems
Sequences

1. Definitions 11.1 #1,2
2. Limit laws (pg 678-680) 11.1 #20, 41
3. L'Hopital's rule 11.1 # 42
4. Squeeze theorem 11.1 # 40
5. Bounded Monotonic 11.1 # 59

Series

1. **Very Important** 11.2 #1, 2
2. Geometric Series 11.2 #13, 20

3. Nth Term test - What, PRECISELY does it say? Can you use it to prove convergence? (No) 11.2 #31
4. Properties of Series (pg 693) 11.2 #26, 28
5. Integral Test - When can you use it? When are you out of luck? 11.3 # 1, 7, 21
6. Remainder Estimates for Integral test 11.3 #36
7. Comparison Test 11.4 # 1, 12
8. Limit Comparison test 11.4 # 20, 24
9. Alternating Series 11.5 # 1, 16
10. Alternating Series Remainder Estimates 11.5 # 24
11. **Absolute vs. conditional convergence** - Know the definitions. Can you have both?
12. Ratio Test 11.6 # 8, 9
13. Root Test 11.6 # 21, 23
14. Read 11.7, its really short!

Power Series

1. Radius and Intervals of Convergence 11.8 # 2, 11, 19
2. Series as Functions
   (a) Integrate/Differentiate 11.9 # 13, 15
   (b) Algebraic manipulation 11.9 # 4
   (c) Finding the sums of series
3. Binomial Series 11.10#35
4. Taylor and Maclaurin Series
   (a) know the common ones (pg 743) 11.10 # 31, 35
   (b) What is the definition of a Taylor Series? A Maclaurin Series? Find a series from this definition 11.10 # 10, 13
   (c) Taylors Remainder formula: make sure you know how to use it to estimate errors and to decide how many terms of a series you need to make sure the error is less than some tolerance. 11.10 # 25, 11.10 #27
   (d) Other Taylor series tricks: evaluating limits 11.11#55, approximating integrals 11.10 #47

Chapter 9

Some Problems:
Solve the following:

1. $y = \sin(x)(1-y)$
2. $y = x + y$
3. $y' = \frac{x^2+2y^2}{3xy}$
4. $y' + x^2 y = 3x$
5. Find the equation for a curve $y(x)$ which at every point (x, y) has a slope equal to $(1+y^2)\sec^2(x)$ and which passes through the point $(0, \pi)$.
6. Suppose you are in charge of a bank. Suppose that people come in at a rate of 100 people per hour and leave at a rate of 50 people per hour. Suppose that on average people have 100 dollars apiece in their wallets when they come in, and that they leave with 10

Conceptual Tasks

1. Ask yourself: what is a differential equation?
2. What is separation of variables?
3. Explain how using an integrating factor to solve a differential equation is related to the product rule for differentiation.

Topics and sample book problems

1. Guess and Check, Initial Value Problems 9.1 # 7
2. Slope Fields 9.2 #2, 3-6
3. Eulers Method 9.2 #23
4. Separable Equations 9.3 # 15
5. Linear Diff Eq/ Integrating Factors 9.5 #

CHAPTER 17

Some Problems

First, decide what method makes sense and come up with a specific strategy (such as a form of undetermined coefficients) for each problem. Then, solve the problems.

1. $y'' + 4y' + 3y = \frac{1}{1+e^{2x}}$
2. $y' + \frac{\ln(x)}{x} y = \frac{\ln(x)}{x}$
3. $y'' - 8y' + 16y = e^{4x}$
4. $y'' + \pi y' + \pi^3 y = 0$
5. $y'' + y = \csc(x)$
6. $y'' - y = \frac{1}{x}$
7. $y' + (1 - x^2)^{-1/2} y = 3$
8. $y'' + 2y' + y = e^x \cos(x)$
9. $y'' + y = e^x + \sec(x)$
10. $y'' + 6y' - y = \sin(at)$

Use a series expansion to solve the following problems:

(a)

$$y'' + x^2 y + xy = 5, \quad y(0) = 0, \quad y'(0) = 1$$

(b)

$$x^2 y'' + xy' + x^2 y - 0, \quad y(0) = 1, \quad y'(0) = 0$$

Conceptual Tasks

1. Ask yourself: what does it mean to be a second order differential equation? A linear one? A homogeneous one? How do those characteristics influence what methods I would use to solve the problem?
2. What is a homogeneous solution? A particular solution? What kind of problems require you to use these, and how do you use them?
3. What is a recursion relation? How do you solve one?

Topics and sample book problems

1. Solve Homogeneous problems, IVPs, BVPs 17.1 #19, 29
2. Characteristic and Auxiliary Equations
3. Method of Undetermined Coefficients 17.2 #3,7
4. Method of Variation of Parameters 17.2 #23
5. Applications of Differential Equations: Springs and pendulums, oh my! 17.3 #10
6. Series Solutions of Differential Equations 17.4 #11