

## 1. STRAIGHTFORWARD PROBLEMS

(1) Consider the series  $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$ . Perform the ratio test for convergence – is it conclusive?

(2) The series  $\sum_{n=1}^{\infty} \frac{1-n}{1+n}$ :

A. converges absolutely. B. converges conditionally. C. diverges.

(3)  $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$ :

A. converges absolutely. B. converges conditionally. C. diverges.

(4) Find the binomial series for  $y = (1 - \frac{x}{2})^{-1/2}$ .

(5) Find the limit of the sequence  $\{a_n\} = \left\{ \frac{n \ln n}{n^2 + 5} \right\}$ .

(6) Which of the following series converge (state which tests/rules you use):

I.  $\sum_{n=1}^{\infty} \frac{1}{n^2}$     II.  $\sum_{n=1}^{\infty} 2^n$     III.  $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$     IV.  $\sum_{n=1}^{\infty} (\frac{1}{2})^n$

## 2. TRICKY PROBLEMS

(1) Find the center and radius of convergence of the power series

$$\sum_{n=2}^{\infty} \frac{(-1)^n(1+n)}{2n} (x-5)^n.$$

Investigate the convergence on the endpoints of the interval.

(2) Do the same for:

$$\sum_{n=2}^{\infty} \frac{2^n(x-3)^n}{\sqrt{n}}.$$

(3) Write the second-degree Taylor polynomial ( $T_2$ ) for  $f(x) = \sqrt{x}$  centered at  $a = 100$ . Use this to estimate  $\sqrt{101}$ . Estimate the error ( $R_2$ ).

(4) Find the first four nonzero terms of the MacLaurin series for  $\int_0^x \sqrt{1+t^3} dt$ .

(5) Find the value of

$$\lim_{x \rightarrow 0^+} \frac{\sin x - x}{2x^3}$$

(6) What is the limit as  $n \rightarrow \infty$  of the sequence

$$\left\{ \left(1 + \frac{1}{n^2}\right)^n \right\}$$

(7) Express these series as closed form functions:

I.  $\sum_{k=0}^{\infty} \frac{x^{k+3}}{3^k k!}$     II.  $\sum_{n=0}^{\infty} \frac{n}{n+1} x^n$     III.  $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2 \cdot 4 \cdot 6 \cdots (2n)}$

## 3. CHALLENGE PROBLEMS

(1) Find the coefficient of  $x^3$  in the MacLaurin series for  $xe^x \cos(x/2)$ .

(2) Does the series  $\sum_{n=2}^{\infty} \frac{\log_n(n!)}{n^3}$  converge or diverge? Explain.

(3) Find the sum of the series

$$\sum_{n=3}^{\infty} \ln\left[\left(\frac{n}{n+1}\right)^3\right]$$

(4) Find  $f^{(5)}(3)$  where  $f(x) = x \ln(x) - 3 \ln(x)$ .

(5) (a) Find the Taylor Series of  $\frac{1}{1-x}$  centered at  $a = -2$ . Find the radius of convergence.

(b) Based on this calculation, evaluate:

$$\sum_{n=0}^{\infty} \frac{-(3-e)^n}{n \cdot 3^n}$$