Quiz 11

Math 1B, Spring 2012

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SOLUTION

Solve the second-order differential equation:

$$y'' + 4y' + 4y = \frac{e^{-2x}}{x^3},$$

using variation of parameters.

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1. We find the complementary solution. The auxiliary equation is:

$$r^2 + 4r + 4 = (r+2)^2 = 0.$$

Therefore, we have a repeated root $r = -2$, so our complementary solution is:

$$y_c = c_1 e^{-2x} + c_2 x e^{-2x}.$$

2. We let $y_p = u_1 e^{-2x} + u_2 x e^{-2x}$.

3. We set up the following system of equations:

$$u_1' e^{-2x} + u_2' x e^{-2x} = 0.$$

$$-2u_1' e^{-2x} + u_2'(-2xe^{-2x} + e^{-2x}) = \frac{e^{-2x}}{x^3}.$$

4. Solving for u_1' and u_2', we obtain:

$$u_1' = -\frac{1}{x^2}, \qquad u_2' = \frac{1}{x^3}.$$

5. Integrating to find u_1 and u_2, we obtain:

$$u_1 = \frac{1}{x}, \qquad u_2 = -\frac{1}{2x^2}.$$

6. Substituting into our original formula for y_p, we find:

$$y_p = \frac{e^{-2x}}{x} - \frac{xe^{-2x}}{2x^2} = \frac{e^{-2x}}{2x}.$$

7. Combining y_c and y_p, our solution is:

$$y(x) = c_1 e^{-2x} + c_2 x e^{-2x} + \frac{e^{-2x}}{2x}.$$