Quiz 9

MATH 1B, SPRING 2012

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SOLUTION

Solve the second-order differential equation:

$$xy'' + 2y' = 12x^2.$$

by making the substitution $u = y'$.

Solution: We substitute $u = y'$ and $u' = y''$. The resulting differential equation is:

$$xu' + 2u = 12x^2 \Rightarrow u' + \frac{2}{x}u = 12x.$$

The integrating factor is $\exp(\int \frac{2}{x}dx) = e^{2\ln x} = x^2$, which gives us:

$$x^2u' + 2xu = 12x^3 \Rightarrow (x^2u)' = 12x^3 \Rightarrow x^2u = \int 12x^3dx$$

$$\Rightarrow u = \frac{1}{x^2}\int 12x^3dx = \frac{1}{x^2}(3x^4 + C) \Rightarrow u = 3x^2 + \frac{C}{x^2}.$$

Finally, we substitute back our original function using the identity $u = y'$:

$$y' = 3x^2 + \frac{C}{x^2} \Rightarrow y = \int 3x^2 + \frac{C}{x^2}dx \Rightarrow y = x^3 - \frac{C}{x} + D.$$

Since C and D are both arbitrary constants, we can also write $y = x^3 + \frac{C}{x} + D$. (This sign change is not strictly necessary.)