(Problems selected from worksheets by Rob Bayer)

(1) **Direction Field Practice.** On the back of the page, there are 4 direction fields.
   (a) Without thinking hardly at all, which one of these is for $y' = 1 + y$? Why?
   (b) The differential equations for the other ones are $y' = x^2 - y^2, y' = y\sin(2x)$, and $y' = 1 - xy$. Determine which is which.
   (c) Using the direction fields, sketch some solution curves to $y' = x^2 - y^2$.

(2) **Separable Equations word problems!**
   (a) A tank initially contains 100L of water with 1000g of salt dissolved in it. Brine containing 50g/L of salt is pumped in at a rate of 2L/min. The solution is kept thoroughly mixed and solution leaves the tank at a rate of 2L/min. Set up and solve an initial value problem whose solution would give you the grams of salt in the tank at time t.
   Hint 1: The rate of change of the amount of salt is the same as (the amount of salt coming in) − (the amount of salt leaving).
   Hint 2: The amount of salt leaving depends on how much salt is in the solution now.
   (b) A certain curve in the plane has the property that every normal line (that is, a line perpendicular to the tangent line) to the curve passes through $(2, 0)$. Find the equation for this curve if you know it passes through $(1, 1)$.
   Hint: What this problem is really asking you is to find a curve where at each point (x, y), the tangent line (which has slope dy/dx) is perpendicular to the line from $(2, 0)$ to (x, y) (what is the slope of this line?).

(3) Consider the differential equation $y' = (y - 3)(y + 2)^2(y + 4)$.
   (a) Without solving for y, what are the equilibrium solutions of this differential equation?
   (b) Sketch a graph with the equilibrium solutions, and other solutions in between. (Consider where the slope is positive or negative.)
   (c) Use separable equations to find an expression for x in terms of y. (y can't be written simply as a function of x.)

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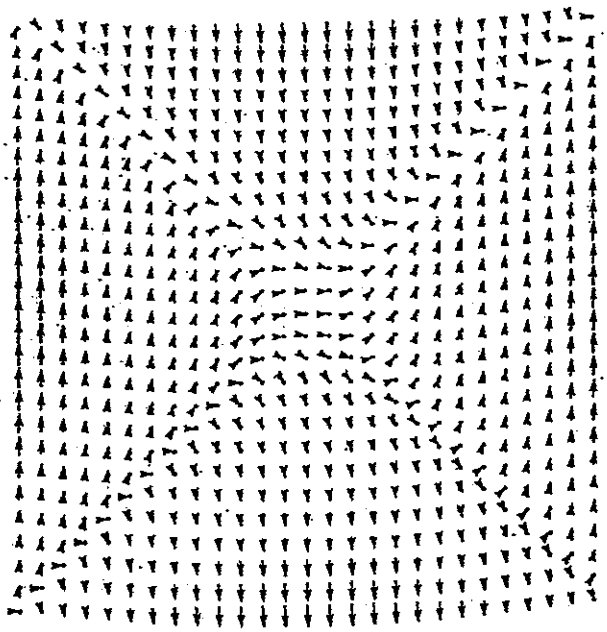

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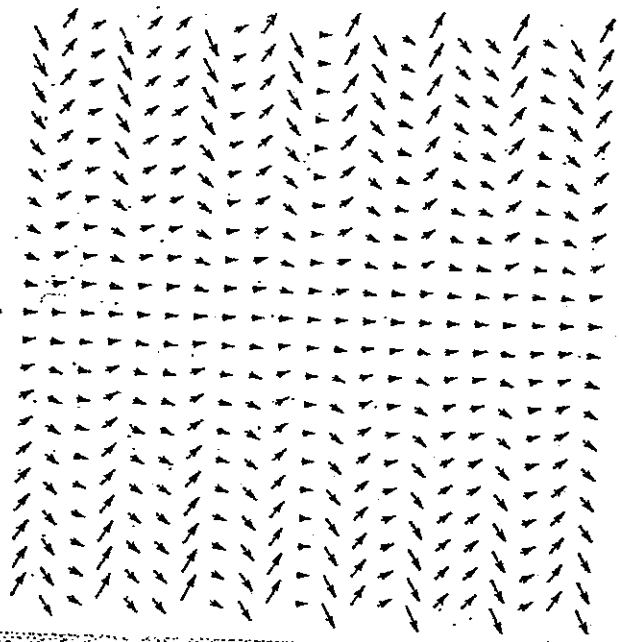

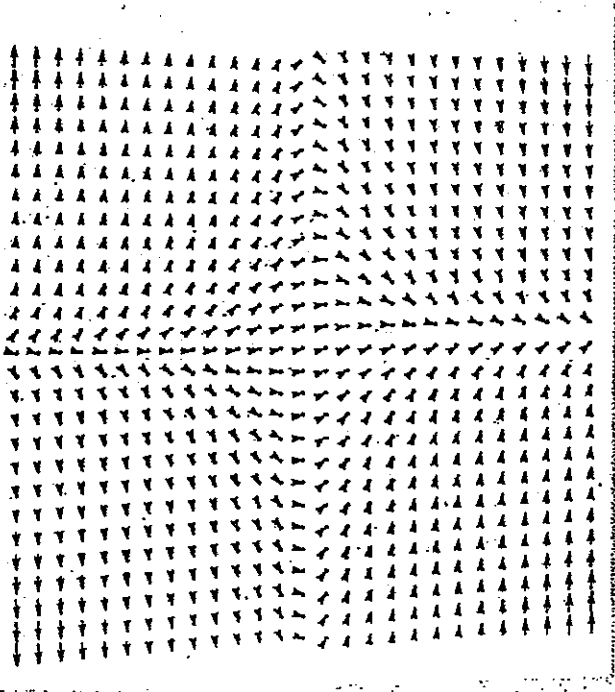

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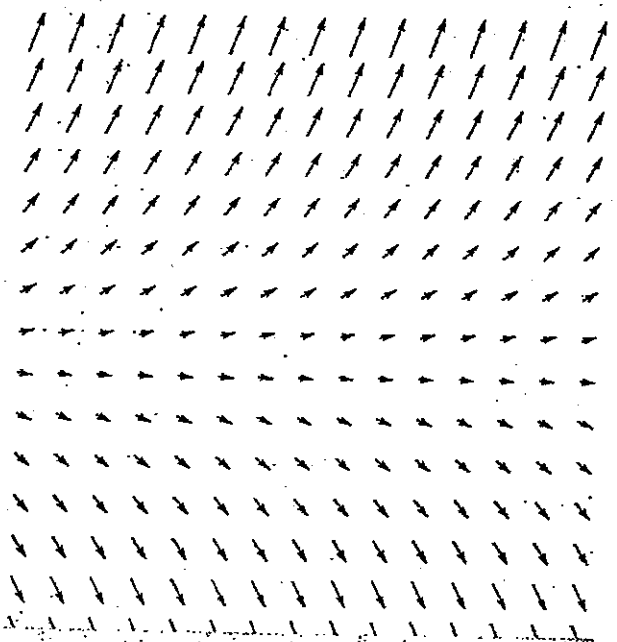

Possibilities:

- $y' = 1 + y$
- $y' = x^2 - y^2$
- $y' = y \sin(2x)$
- $y' = 1 - xy$