Please submit your answers typed up in TeX, either in hard copy or via e-mail by the beginning of class on February 13.

1. **(10 points)** Suppose that you want to walk from $(0,0,0)$ to $(5,5,5)$ in a cubic grid traveling exactly 15 unit steps. You are additionally required to hit the point $(1,2,3)$ and avoid the point $(3,4,5)$. How many possible paths are there satisfying these requirements? Justify your answer.

2. **(20 points)** (EC1, Problem 1.5) Show that

$$\sum_{n_1,\ldots,n_k \geq 0} \min(n_1,\ldots,n_k) x_1^{n_1} \cdots x_k^{n_k} = \frac{x_1 \cdots x_k}{(1-x_1)\cdots(1-x_k)(1-x_1x_2\cdots x_k)}.$$

3. **(20 points)** (EC1, Problem 1.26) Let $\bar{c}(m,n)$ denote the number of compositions of n with largest part at most m. Show that

$$\sum_{n\geq 0} \bar{c}(m,n)x^n = \frac{1-x}{1-2x+x^{m+1}}.$$

4. **(20 points)** (From Prof. Mark Haiman) Given that 2020 is an election year in the US, find a formula for the generating function $\sum a_n x^n$ where a_n is the number of combinations of states and the District of Columbia having a combined number n of electoral votes. You may ignore the fact that Maine and Nebraska split their electors.

   Using a computer algebra system like `Sage` or `Macaulay2`, evaluate the generating function explicitly and answer the following question: in a presidential election with two candidates, how many combinations lead to a tie in the electoral college? Supposing that all outcomes are equally likely, what is the probability of a tie?

   Explain your answer, and include your code.

5. **(30 points)** (From Prof. Mark Haiman) If $F(x)$ and $G(x)$ are formal power series in $R[[x]]$ (i.e. the ring of formal power series with coefficients in the ring R) and $F(0) = 0$, i.e. $F(x)$ has zero constant term, their formal composition is defined by $(G \circ F)(x) = \sum_{k=0}^{\infty} g_k F(x)^k$.

   (a) Show that composition is associative, i.e. if $F(0) = 0$ and $G(0) = 0$ then $(H \circ G) \circ F = H \circ (G \circ F)$.

   (b) Show that $F(x) \in R[[x]]$ such that $F(0) = 0$ has a formal compositional inverse if and only if the coefficient $[x]F(X)$ has a multiplicative inverse in R.