Analytic Combinatorics, ECCO 2012
Exercise Sheet 1

1. Explain why $n! \sim \left(\frac{n}{e}\right)^n$.
2. Deduce from the Stirling formula that 100! has 158 digits. (1000)!?
3. Find all alternating permutations $n = 5$.
4. Use Sloan to find T_{17}.
5. Prove that $T_n =$ number of labeled binary decreasing trees.
6. Explain why $T(z) = \sum_{n \geq 0} T_n \frac{z^n}{n!}$ is a solution of $\frac{dT(z)}{dz} = 1 + T(z)^2$.
7. Check that $T(z) = \tan(z)$.
8. Explain (prove) that for n odd,
$$T_n - \binom{n}{2} T_{n-2} + \binom{n}{4} T_{n-4} - \cdots = (-1)^{\frac{n-1}{2}}.$$
9. (a) Show that $\tan(z) \sim \dfrac{8z}{\pi^2 - 4z^2}$.

   (b) Show that $\dfrac{T_n}{n!} \sim 2 \cdot \left(\dfrac{2}{\pi}\right)^{n+1}$.
10. Prove that the ordinary generating function $C(z) = \sum c_n z^n$ for binary trees on $n+3$ vertices satisfies
$$C(z) = 1 + C(z)^2.$$