Analytic Combinatorics, ECCO 2012
Exercise sheet 4

1. Let $h(z)$ be a power series with radius of convergence $r > 1$. Assume that $h(1) \neq 0$. Show that

   (a)
   $$[z^n]\frac{h(z)}{1-z} \sim h(1),$$

   (b)
   $$[z^n]h(z)\sqrt{1-z} \sim \frac{h(1)}{2\sqrt{\pi n^3}},$$

   (c)
   $$[z^n]h(z)\log(\frac{1}{1-z}) \sim \frac{h(1)}{n}.$$

2. From the double surjection problem we got that $R^*(z) = (2 + z - e^z)$. Denote by ρ^* its dominant pole.

   (a) Approximate $R^*(z)$ near its dominant pole.

   (b) Approximate $[z^n]R^*(z)$.

3. Let $O(z)$ the exponential generating function of $\mathcal{O} = SEQ(CYC(\mathcal{Z}))$. Approximate $O(z)$ at its dominant singularity.